Performance estimation of first-order methods for composite convex optimization

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Composite convex optimization consists in the minimization of a convex objective function equal to the sum of a smooth term and a nonsmooth term, over a given convex feasible region. We consider a large class of first-order methods designed to solve such composite problems, which rely on specific oracles for each term. We show that the worst-case performance of each of those methods can be computed exactly by solving a semidefinite optimization problem, which also produces an explicit problem instance for which this worst-case is attained.

The performance estimation methodology was born in the original work of Drori [1], which introduced a semidefinite relaxation of the problem to improve upon several classical convergence results for several standard first-order optimization algorithms for smooth unconstrained convex optimization. In this work, we present the extension developed in [2, 3], which produces tight convergence bounds in the case of *fixed-step linear first-order methods* [3] for composite convex optimization. These include classical and accelerated gradient methods (including constrained and proximal variants), conditional gradient and subgradient methods, and inexact gradient computations.

In particular, we improve the standard convergence bounds for the proximal point algorithm by a factor of two, and improve that of the conditional gradient method by a similar factor. We also propose an extension of the optimized gradient method proposed by Kim and Fessler in [4] to handle a projection or a proximal operator, which is in the worst-case twice better than the standard accelerated proximal gradient method [5].

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